What You’ll Learn

Key Ideas

• Identify and use parts of circles. (Lesson 11–1)

• Identify major arcs, minor arcs, and semicircles and find the measures of arcs and central angles. (Lesson 11–2)

• Identify and use the relationships among arcs, chords, and diameters. (Lesson 11–3)

• Inscribe regular polygons in circles and explore lengths of chords. (Lesson 11–4)

• Solve problems involving circumferences, areas, and sectors of circles. (Lessons 11–5 and 11–6)

Key Vocabulary

center (p. 454)
circle (p. 454)
circumference (p. 478)
diameter (p. 454)
radius (p. 454)

Why It’s Important

Navigation  Latitude and longitude and the system for finding your position at any time were developed in order to allow safe travel by ships at sea. Horizontal circles called latitude lines indicate distance north or south of the equator. Meridians, or circles that pass through both poles, determine longitude.

Circles are used in sports, biology, carpentry, and other fields. You will investigate the measures of angles between the meridians in Lesson 11–2.
Study these lessons to improve your skills.

Refer to the line for Exercises 1–4.

1. If \( LM = 8 \) and \( LP = 22 \), find \( MP \).
2. If \( NP = 6 \) and \( PQ = 10 \), find \( NQ \).
3. If \( LN = 16 \) and \( LM = 8 \), find \( MN \).
4. If \( PR = 12 \) and \( LP = 22 \), find \( LR \).

Refer to the figure at the right.

5. If \( m\angle AOB = 55 \) and \( m\angle BOC = 32 \), find \( m\angle AOC \).
6. Find \( m\angle DOB \) if \( m\angle AOB = 55 \) and \( m\angle AOD = 130 \).
7. If \( m\angle EOD = 50 \) and \( m\angle COE = 93 \), find \( m\angle DOC \).

If \( c \) is the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

8. \( a = 9 \), \( b = 12 \), \( c = ? \)
9. \( a = ?, b = 15 \), \( c = 17 \)
10. \( a = 40 \), \( b = ?, c = 41 \)
11. \( a = 8 \), \( b = 14 \), \( c = ? \)

Make this Foldable to help you organize your Chapter 11 notes. Begin with seven sheets of \( 8\frac{1}{2} \)“ by 11” paper.

1. Draw and cut a circle from each sheet. Use a small plate or CD to outline the circles.

2. Staple the circles together to form a booklet.

3. Label the front of the booklet Circles. Label the six inside pages with the lesson titles.

Reading and Writing As you read and study the chapter, use each page to write definitions and theorems and draw models. Also include any questions that need clarifying.
A circle is a special type of a geometric figure. All points on a circle are the same distance from a center point.

The measures of $\overline{OA}$ and $\overline{OB}$ are the same; that is, $OA = OB$.

There are three kinds of segments related to circles. A radius is a segment whose endpoints are the center of the circle and a point on the circle. A chord is a segment whose endpoints are on the circle. A diameter is a chord that contains the center.

From the figures, you can note that the diameter is a special type of chord that passes through the center.
Use \( \odot Q \) to determine whether each statement is true or false.

1. \( \overline{AD} \) is a diameter of \( \odot Q \).
   
   False; \( \overline{AD} \) does not go through the center \( Q \). Thus, \( \overline{AD} \) is not a diameter.

2. \( \overline{BQ} \) is a radius of \( \odot Q \).
   
   True; the endpoints of \( \overline{BQ} \) are the center \( Q \) and a point on the circle \( B \). Thus, \( \overline{BQ} \) is a radius.

**Your Turn**

a. \( \overline{AC} \) is a chord of \( \odot Q \).

b. \( \overline{AD} \) is a radius of \( \odot Q \).

Suppose the radius \( \overline{PQ} \) of \( \odot P \) is 5 centimeters long. So, the radius \( \overline{PT} \) is also 5 centimeters. Then the diameter \( \overline{QT} \) is 5 + 5 or 10 centimeters long. Notice that the diameter is twice as long as the radius. This leads to the next two theorems.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Words</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>11–1</td>
<td>All radii of a circle are congruent.</td>
<td>( \overline{PR} \cong \overline{PQ} \cong \overline{PS} \cong \overline{PT} )</td>
</tr>
<tr>
<td>11–2</td>
<td>The measure of the diameter ( d ) of a circle is twice the measure of the radius ( r ) of the circle.</td>
<td>( d = 2r ) or ( \frac{1}{2}d = r )</td>
</tr>
</tbody>
</table>

**Example**

3. In \( \odot T \), \( \overline{CD} \) is a diameter. If \( CD = 42 \), find \( TC \).

   \( \overline{TC} \) is a radius of \( \odot T \).

\[
\begin{align*}
  d &= 2r & \text{Definition of radius} \\
  CD &= 2(\overline{TC}) & \text{Replace } d \text{ with } CD \text{ and } r \text{ with } TC. \\
  42 &= 2(\overline{TC}) & \text{Replace } CD \text{ with } 42. \\
  \frac{42}{2} &= \frac{2(\overline{TC})}{2} & \text{Divide each side by } 2. \\
  21 &= \overline{TC} & \text{Simplify.}
\end{align*}
\]

**Your Turn**

c. In \( \odot T \), \( \overline{TM} \) is a radius. If \( TM = 15.5 \), find \( CD \).
Find the measure of radius $PC$ if $PC = 2x$ and $AB = 6x - 28$.

\[
d = 2r
\]
\[
AB = 2(PC)
\]
Replace $d$ with $AB$ and $r$ with $PC$.

\[
6x - 28 = 2(2x)
\]
Replace $AB$ with $6x - 28$ and $PC$ with $2x$.

\[
6x - 28 = 4x
\]
Simplify.

\[
6x - 28 - 6x = 4x - 6x
\]
Subtract $6x$ from each side.

\[
-28 = -2x
\]
Simplify.

\[
\frac{-28}{-2} = \frac{-2x}{-2}
\]
Divide each side by $-2$.

\[
14 = x
\]
Simplify.

$PC = 2(14)$ or 28

Because all circles have the same shape, any two circles are similar. However, two circles are congruent if and only if their radii are congruent. Two circles are **concentric** if they meet the following three requirements.

- They lie in the same plane.
- They have the same center.
- They have radii of different lengths.

**Check for Understanding**

**Communicating Mathematics**

1. Explain why there are more than two radii in every circle. How many radii are there?

2. Describe how to find the measure of the radius if you know the measure of the diameter.

3. Jason says that every diameter of a circle is a chord. Amelia says that every chord of a circle is a diameter. Who is correct, and why?

**Guided Practice**

**Getting Ready**

If $r$ is the measure of the radius and $d$ is the measure of the diameter, find each measure.

**Sample:** $d = 9.04, r = \_?\_$$$

**Solution:** $r = \frac{9.04}{2}$ or 4.52

4. $r = 3.8, d = \_?\_$$$

5. $d = 3\frac{1}{2}, r = \_?\_$$$

6. $r = \frac{x}{2}, d = \_?\_$$$

**Vocabulary**

- circle
- center
- radius
- chord
- diameter
- concentric
Examples 1 & 2  Use $\odot F$ to determine whether each statement is true or false.

7. $FD$ is a radius of $\odot F$.
8. $AB$ is a diameter of $\odot F$.
9. $CE$ is a chord of $\odot F$.

Example 3  Use $\odot F$ to complete the following.

10. If $CE = 15.2$, find $FD$.
11. If $FE = 19$, find $CE$.

Example 4  12. **Algebra** Find the value of $x$ in $\odot K$.

---

**Exercises**

Use $\odot R$ to determine whether each statement is true or false.

13. $HB$ is a radius of $\odot R$.
14. $HD = 2(RD)$
15. $CG$ is a diameter of $\odot R$.
16. $BE$ is a diameter of $\odot R$.
17. $RE$ is a chord of $\odot R$.
18. $RC$ is a radius of $\odot R$.
19. $AF$ is a chord of $\odot R$.
20. $RH = RG$

21. A circle has exactly two radii.
22. A radius of a circle is a chord of the circle.

$\odot P$ has a radius of 5 units, and $\odot T$ has a radius of 3 units.

23. If $QR = 1$, find $RT$.
24. If $QR = 1$, find $PQ$.
25. If $QR = 1$, find $AB$.
26. If $AR = 2x$, find $AP$ in terms of $x$.
27. If $TB = 2x$, find $QB$ in terms of $x$.

Applications and Problem Solving  28. **Music** Most music compact discs (CDs) have three concentric circles. The first circle forms the hole in the CD and has a diameter of 2.5 centimeters. The second circle forms an inner ring, on which no data are stored. It has a diameter of 4 centimeters. The third circle forms the disc itself. It has a diameter of 12 centimeters. What are the radii of the three circles?
29. **Algebra** In \( S, VK = 3x - 9 \) and \( JW = 2x + 15 \). Find the measure of a radius of \( S \).

30. **Sports** Identify the types of circles shown below as congruent, concentric, or similar.

![Archery target](image1.png)  ![Old-style bicycle](image2.png)  ![Olympic rings](image3.png)

a. archery target  
b. old-style bicycle  
c. Olympic rings

31. **Critical Thinking** Give a reason for each statement to show that a diameter is the longest chord of a circle. Assume that \( T \) is the center.

a. \( QT + TR > QR \)  
b. \( QT = AT \) and \( TR = TB \)  
c. \( AT + TB > QR \)  
d. \( AB > QR \)

32. Identify the figures used to create the tessellation. Then identify the tessellation as regular, semi-regular, or neither. (Lesson 10–7)

33. Does the figure have line symmetry? rotational symmetry? (Lesson 10–6)

34. \( FD \)  
35. \( EF \)  
36. \( CG \)

37. **Extended Response** The caveman is using inductive reasoning to make a conjecture. (Lesson 1–1)

a. List the steps he uses to make his conjecture.  
b. State the conjecture.

38. **Multiple Choice** (Algebra Review) Simplify \( \frac{\sqrt{6} \cdot \sqrt{8}}{\sqrt{3}} \).

- A. \( \sqrt{4} \)  
- B. 4  
- C. 16  
- D. 48

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“Water boils down to nothing... snow boils down to nothing... ice boils down to nothing... everything boils down to nothing.”

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Woodworker

Frequently, a woodworker who makes furniture will need to cut a circular table top from a square piece of wood stock, as shown at the right. In order to cut the table top, the woodworker first needs to locate the center of the wood. Then a radius can be found and a circle can be drawn.

One method for finding the center is shown below.

Step 1  Make sure that the piece of stock is square.
Step 2  Use a ruler and pencil to draw the diagonals of the square.
Step 3  Mark the point at which the diagonals meet.
Step 4  Use a thumbtack, pencil, and string to sketch the circle. The radius of the circle will be one-half the length of a side of the square.

1. Sketch the largest circle that will fit inside a square 2 inches on a side.
2. Sketch the largest circle that will fit inside an equilateral triangle \(1\frac{1}{2}\) inches on a side.
3. The table top shown will have a mahogany inlay in the shape of a circle. The inlay will be placed in the center of the rectangle and will have a diameter that is one-half the width of the rectangle. Copy the rectangle. Then sketch the circle described.

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**FAST FACTS About Woodworkers**

**Working Conditions**
- production woodworkers usually work in large plants on mass-produced items
- precision woodworkers usually work in small shops on one-of-a-kind items
- woodworkers often wear eye and ear protection for safety

**Education**
- most are trained on the job; may require two or more years of training
- high school diploma desired
- ability to pay attention to detail a must

**Earnings**

*Weekly Earnings for Cabinetmakers and Bench Carpenters*

<table>
<thead>
<tr>
<th></th>
<th>Lowest 10%—less than $290</th>
<th>Middle 50%—between $348 and $549</th>
<th>Top 10%—more than $688</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td></td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

Median earnings: about $433 per week

*Source:* Enhanced Occupational Outlook Handbook

Career Update  For the latest information about a career as a woodworker, visit: www.geomconcepts.com
Investigation

A Locus Is Not a Grasshopper!

Loci

A **locus** is the set of all points that satisfy a given condition or conditions. The plural of locus is **loci** (LOW-sigh). You have already seen some sets of points that form loci. Let's investigate this idea further.

Investigate

1. Given a point $A$, what is the locus of points in a plane that are two feet from point $A$?

   a. Cut a square piece of cardboard with side length of 3 inches. Cut a piece of string about 30 inches in length. Poke a small hole in the center of the cardboard. Label the small hole $A$. Thread the string through the hole. Tie a knot in the string and cut it so that it measures two feet from point $A$ to the end of the string.

   b. Tape the cardboard square securely to the floor with the knot under the cardboard. Pull the string to its full length along the floor. Mark the floor at the end of the string with a colored dot. The dot will represent a point that is two feet from point $A$.

   c. Pick the string up and extend it in another direction. Mark the floor at the end of the string with another colored dot. Repeat this process until you have about 20 dots on the floor.

   d. Imagine placing more and more dots on the floor. Describe this set of points.

   e. Describe the locus of points in a plane at a given distance from a fixed point on the plane.
2. Given two parallel lines, what is the locus of points in a plane that are equidistant from the two parallel lines?
   a. Tape two metersticks to the floor so that they are parallel and at least one foot apart. Make sure that the ends of the metersticks are even.
   b. Cut a piece of string the length of the distance between the metersticks. To be accurate, stretch the string between the same markings on the sticks, say between 10 centimeters on each stick.
   c. Find and mark the midpoint of the piece of string. Lay the string between the same markings on the two sticks. Place a colored dot on the floor at the midpoint of the string. Pick up the string and mark about 10 additional points with colored dots.
   d. Imagine placing more and more dots. Describe this set of points.
   e. Describe the locus of points in a plane that are equidistant from the two parallel lines.

1. Identify the locus of points that satisfies each condition.
   a. all points in a plane that are 2 feet from a given line
   b. all points in a plane that are equidistant from two given points
   c. all points in a plane that are equidistant from the sides of a given angle
   d. all points in a plane that are equidistant from two intersecting lines

2. A **compound locus** is the intersection of loci that satisfies two or more conditions. Identify the compound locus of points that satisfies each set of conditions.
   a. all points in a plane that are equidistant from the sides of a given 90° angle and 4 feet from the vertex of the angle
   b. all points in a plane that are equidistant from two given points and 10 centimeters from the line containing the two points
   c. all points on a coordinate plane that are 2 units from \( P(1, 3) \) and 1 unit from \( Q(1, 6) \)

3. If the phrase “in a plane” was deleted from the two problems in the Investigation, what would the locus be for those two problems? Provide a sketch.

**Presenting Your Conclusions**

Here are some ideas to help you present your conclusions to the class.

- Make a poster with scale drawings and descriptions of the methods and materials you used to find the solution to each locus problem.
- Make a three-dimensional display using marbles for points, straws to show distance, and dowels for line segments or sides of angles.

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**Investigation**  For more information on loci, visit: www.geomconcepts.com
A central angle is formed when the two sides of an angle meet at the center of a circle. Each side intersects a point on the circle, dividing it into arcs that are curved lines.

There are three types of arcs. A minor arc is part of the circle in the interior of the central angle with measure less than 180°. A major arc is part of the circle in the exterior of the central angle. Semicircles are congruent arcs whose endpoints lie on a diameter of the circle.

Arcs are named by their endpoints. Besides the length of an arc, the measure of an arc is also related to the corresponding central angle.

Note that for \( \odot A \), two letters are used to name the minor arc, but three letters are used to name the major arc and semicircle. These letters for naming arcs help us trace the set of points in the arc. In this way, there is no confusion about which arc is being considered.

Depending on the central angle, each type of arc is measured in the following way.
Definition of Arc Measure

1. The degree measure of a minor arc is the degree measure of its central angle.
2. The degree measure of a major arc is 360 minus the degree measure of its central angle.
3. The degree measure of a semicircle is 180.

Example

In $\odot R$, $KN$ is a diameter. Find $m\widehat{ON}$, $m\angle NRT$, $m\angle OTK$ and $m\angle NTK$.

$m\widehat{ON} = m\angle ORN$  
Measure of minor arc  
$= 42$  
Substitution

$m\angle NRT = m\widehat{NT}$  
Measure of central angle  
$= 89$  
Substitution

$m\angle OTK = 360 - m\angle ORK$  
Measure of major arc  
$= 360 - 138$  
Substitution  
$= 222$  
Subtract.

$m\angle NTK = 180$  
Measure of semicircle

Note that the sum of the measures of the central angles of $\odot R$ is 360.

Your Turn

a. In $\odot P$, find $m\widehat{AM}$, $m\angle APT$, and $m\angle THM$.

In $\odot P$ above, $\widehat{AM}$ and $\widehat{AT}$ are examples of adjacent arcs. Adjacent arcs have exactly one point in common. For $\widehat{AM}$ and $\widehat{AT}$, the common point is $A$. The measures of adjacent arcs can also be added.

Postulate 11–1
Arc Addition Postulate

Words: The sum of the measures of two adjacent arcs is the measure of the arc formed by the adjacent arcs.

Model:  
Symbols: If $Q$ is a point on $\widehat{PR}$, then $m\widehat{PQ} + m\widehat{QR} = m\widehat{PQR}$.
Example 2

In \( \odot P \), \( RT \) is a diameter. Find \( m_{RS}, m_{ST}, m_{STR} \), and \( m_{QS} \).

\[
m_{RS} = m \angle RPS \quad \text{Measure of minor arc}
\]
\[
= 65 \quad \text{Substitution}
\]

\[
m_{RS} + m_{ST} = m_{RST} \quad \text{Arc addition postulate}
\]
\[
m_{ST} = m_{RST} - m_{RS}
\]
\[
= 180 - 65 \text{ or } 115 \quad \text{Substitution}
\]

\[
m_{STR} = 360 - m \angle RPS \quad \text{Measure of major arc}
\]
\[
= 360 - 65 \text{ or } 295 \quad \text{Substitution}
\]

\[
m_{QS} = m_{QR} + m_{RS} \quad \text{Arc addition postulate}
\]
\[
= 75 + 65 \text{ or } 140 \quad \text{Substitution}
\]

Your Turn

b. In \( \odot P \), find \( m_{QT} \).

c. In \( \odot P \), find \( m_{STQ} \).

Suppose there are two concentric circles with \( \angle ASD \) forming two minor arcs, \( BC \) and \( AD \). Are the two arcs congruent?

\[
m_{BC} = m \angle BSC \text{ or } 60
\]

\[
m_{AD} = m \angle ASD \text{ or } 60
\]

Although \( BC \) and \( AD \) each measure 60, they are not congruent. The arcs are in circles with different radii, so they have different lengths. However, in a circle, or in congruent circles, two arcs are congruent if they have the same measure.

Theorem 11–3

Words: In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.

Model:

Symbols: \( WX \equiv YZ \) if and only if \( m \angle WXQ = m \angle YQZ \).
In \( \odot M \), \( \overline{WS} \) and \( \overline{RT} \) are diameters, \( m\angle WMT = 125 \), and \( m\angle RK = 14 \). Find \( m\overline{RS} \).

\[
\begin{align*}
\angle WMT & \equiv \angle RMS & \text{Vertical angles are congruent.} \\
m\angle WMT & = m\angle RMS & \text{Definition of congruent angles} \\
m\overline{WT} & = m\overline{RS} & \text{Theorem 11–3} \\
125 & = m\overline{RS} & \text{Substitution}
\end{align*}
\]

**Your Turn**

\begin{align*}
d. & \text{ Find } m\overline{KS}. \\
e. & \text{ Find } m\overline{ST}.
\end{align*}

---

**Check for Understanding**

**Communicating Mathematics**

1. Refer to \( \odot R \) with diameter \( \overline{PH} \).

\[
\begin{align*}
a. & \text{ Explain how to find } m\overline{PNH}. \\
b. & \text{ Determine whether } \overline{PNH} \equiv \overline{PHN}. \text{ Explain.} \\
c. & \text{ Explain how to find } m\overline{HPN} \text{ if } m\angle NRH = 35. \\
d. & \text{ Explain why diameter } \overline{PH} \text{ creates two arcs that measure 180.}
\end{align*}
\]

2. **Compare and contrast** minor arcs and major arcs.

3. **You Try It** Marisela says that two arcs can have the same measure and still not be congruent. Dexter says that if two arcs have the same measure, then they are congruent. Who is correct, and why?

**Guided Practice**

**Getting Ready** Determine whether each arc is a minor arc, major arc, or semicircle of \( \odot M \).

**Sample:** \( \overline{DAB} \)

**Solution:** \( m\overline{DAB} > 180 \), so \( \overline{DAB} \) is a major arc.

4. \( \overline{BDE} \)
5. \( \overline{ECA} \)
6. \( \overline{AB} \)
Find each measure in $\odot P$ if $m \angle APB = 30$ and $AC$ and $BD$ are diameters.

7. $m \overline{AB}$
8. $m \overline{ACB}$
9. $m \overline{BAC}$
10. $m \overline{BC}$

11. $m \overline{AD}$

12. **Food** Rosati’s Pizza cuts their pizzas along four diameters, which separate each pizza into eight congruent pieces. What is the measure of the central angle of each piece?

In $\odot Q$, $\overline{AC}$ is a diameter and $m \angle CQD = 40$. Determine whether each statement is true or false.

25. $m \angle CBD = 140$
26. $m \angle CQD = m \overline{CD}$
27. $m \angle ABD$ is a central angle.
28. $m \overline{AD} = 320$
29. $m \overline{ACD} = 140$

$A$ is the center of two circles with radii $\overline{AQ}$ and $\overline{AR}$. If $m \angle SAR = 32$ and $m \overline{XQ} = 112$, find each measure.

30. $m \overline{SR}$
31. $m \angle RAW$
32. $m \overline{WR}$
33. $m \overline{TQ}$
34. $m \overline{TYX}$
35. $m \overline{SZW}$

**Applications and Problem Solving**

36. **Employment** Twenty-two percent of all teens ages 12 through 17 work either full- or part-time. The circle graph shows the number of hours they work per week. Find the measure of each central angle.

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Source: ICRs TeenEXCEL survey for Merrill Lynch
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37. **Geography** Earth has 24 time zones, each of which is centered on a line called a *meridian*.

   a. What is the measure of the arc between 6 P.M. and 5 P.M.?

   b. What is the measure of the minor arc between 6 P.M. and 4 A.M.?

38. **Critical Thinking** In \( \triangle BPR \equiv \triangle QSR \). Show that \( PQ \parallel RS \). Give a reason for each step of your argument.

### Mixed Review

39. **Basketball** Basketball rims are 18 inches in diameter. What is the radius of a rim? *(Lesson 11–1)*

40. Create your own tessellation using squares and triangles. *(Lesson 10–7)*

41. Solve \( \frac{3x - 5}{4} = \frac{x}{2} \). *(Lesson 9–1)*

42. Use a straightedge to draw a quadrilateral that has exactly one diagonal in its interior. *(Lesson 8–1)*

43. **Grid In** The brace shown at the right is used to keep a shelf perpendicular to the wall. If \( m\angle AHM = 40 \), find \( m\angle HAT \). *(Lesson 7–2)*

44. **Short Response** Explain how you could use translations to draw a cube. *(Lesson 5–3)*

### Quiz 1

Use \( \Theta P \) to determine whether each statement is true or false. *NL and MK are diameters of \( \Theta P \).* *(Lessons 11–1 & 11–2)*

1. \( JP \) is a radius.
2. \( JK \) is a radius.
3. \( NP \) is a chord.
4. \( NL = 2(NP) \)
5. \( m\angle M = 54 \)
6. \( m\angle KL = 336 \)
7. \( m\angle NM = 24 \)
8. \( m\angle ML = 126 \)
9. \( m\angle JPK = 102 \)

10. **Time** The hands of a clock form central angles. What is the approximate measure of the central angle at 6:00? at 12:05? at 6:05? *(Lesson 11–2)*

[Diagram of clock with labeled angles]
In circle $P$ below, each chord joins two points on a circle. Between the two points, an arc forms along the circle. By Theorem 11–3, $\overline{AD}$ and $\overline{BC}$ are congruent because their corresponding central angles are vertical angles, and therefore congruent ($\angle APD \cong \angle BPC$).

Can we show that $\overline{AD} \cong \overline{BC}$? You can prove that the two triangles $\triangle APD$ and $\triangle CPB$ are congruent by SAS. Therefore, $\overline{AD}$ and $\overline{BC}$ are congruent.

The following theorem describes the relationship between two congruent minor arcs and their corresponding chords.

**Theorem 11–4**

**Words:** In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

**Symbols:** $\overline{AD} \cong \overline{BC}$ if and only if $\overline{AD} \cong \overline{BC}$.

**Example 1**

The vertices of equilateral triangle $JKL$ are located on $\odot P$. Identify all congruent minor arcs.

Since $\triangle JKL$ is equilateral, $\overline{JK} \cong \overline{KL} \cong \overline{JL}$. By Theorem 11–4, we know that $\overline{JK} \cong \overline{KL} \cong \overline{JL}$.

**Your Turn**

a. The vertices of isosceles triangle $XYZ$ are located on $\odot R$. If $\overline{XY} \cong \overline{YZ}$, identify all congruent arcs.
You can use paper folding to find a special relationship between a diameter and a chord of a circle.

**Hands-On Geometry**

**Paper Folding**

**Materials:**
- compass
- patty paper
- straightedge

**Step 1** Use a compass to draw a circle on a piece of patty paper. Label the center $P$. Draw a chord that is not a diameter. Label it $EF$.

**Step 2** Fold the paper through $P$ so that $E$ and $F$ coincide. Label this fold as diameter $GH$.

**Try These**

1. When the paper is folded, compare the lengths of $EG$ and $FG$. Then compare the lengths of $EH$ and $FH$.
2. What is the relationship between diameter $GH$ and chord $EF$?
3. Make a conjecture about the relationship among a diameter, a chord, and its arc if the diameter is perpendicular to the chord.

This activity suggests the following theorem.

**Theorem 11–5**

**Words:** In a circle, a diameter bisects a chord and its arc if and only if it is perpendicular to the chord.

**Model:**

**Symbols:** $\overline{AR} \cong \overline{BR}$ and $\overline{AD} \cong \overline{BD}$ if and only if $\overline{CD} \perp \overline{AB}$.

Like an angle, an arc can be bisected.
Recall that the distance from a point to a segment is measured on the perpendicular segment drawn from the point to the segment.

In \( \odot P \), if \( PM \perp AT \), \( PT = 10 \), and \( PM = 8 \), find \( AT \).

\( \angle PMT \) is a right angle. \hspace{1cm} \text{Def. of perpendicular}

\( \Delta PMT \) is a right triangle. \hspace{1cm} \text{Def. of right triangle}

\[(MT)^2 + (PM)^2 = (PT)^2\] \hspace{1cm} \text{Pythagorean Theorem}

\[(MT)^2 + 8^2 = 10^2\] \hspace{1cm} \text{Replace \( PM \) with 8 and \( PT \) with 10.}

\[(MT)^2 + 64 = 100\] \hspace{1cm} 8\(^2\) = 64; 10\(^2\) = 100

\[(MT)^2 + 64 - 64 = 100 - 64\] \hspace{1cm} \text{Subtract 64 from each side.}

\[(MT)^2 = 36\] \hspace{1cm} \text{Simplify.}

\[\sqrt{(MT)^2} = \sqrt{36}\] \hspace{1cm} \text{Take the square root of each side.}

\[MT = 6\] \hspace{1cm} \text{Simplify.}

By Theorem 11–5, \( PM \) bisects \( AT \). Therefore, \( AT = 2(MT) \). So, \( AT = 2(6) \) or 12.

In \( \odot R \), \( XY = 30 \), \( RX = 17 \), and \( RZ \perp XY \). Find the distance from \( R \) to \( XY \).

The measure of the distance from \( R \) to \( XY \) is \( RZ \). Since \( RZ \perp XY \), \( RZ \) bisects \( XY \), by Theorem 11–5. Thus, \( XZ = \frac{1}{2}(30) \) or 15.

For right triangle \( RZX \), the following equation can be written.

\[(RZ)^2 + (XZ)^2 = (RX)^2\] \hspace{1cm} \text{Pythagorean Theorem}

\[(RZ)^2 + 15^2 = 17^2\] \hspace{1cm} \text{Replace \( XZ \) with 15 and \( RX \) with 17.}

\[(RZ)^2 + 225 = 289\] \hspace{1cm} 15\(^2\) = 225; 17\(^2\) = 289

\[(RZ)^2 + 225 - 225 = 289 - 225\] \hspace{1cm} \text{Subtract 225 from each side.}

\[(RZ)^2 = 64\] \hspace{1cm} \text{Simplify.}

\[\sqrt{(RZ)^2} = \sqrt{64}\] \hspace{1cm} \text{Take the square root of each side.}

\[RZ = 8\] \hspace{1cm} \text{Simplify.}

The distance from \( R \) to \( XY \), or \( RZ \), is 8 units.

**Your Turn**

Find each measure in each \( \odot K \).

b. \( AB \)

c. \( KM \)
Example

In \( \bigcirc Q \), \( KL \cong LM \). If \( CK = 2x + 3 \) and \( CM = 4x \), find \( x \).

Since \( KL \cong LM \), \( QL \) bisects \( KM \). So, by Theorem 11–5, \( QL \) also bisects \( KM \).

Thus, \( CM \cong CK \).

\[
CM = CK \\
4x = 2x + 3 \\
4x - 2x = 2x + 3 - 2x \\
2x = 3 \\
\frac{2x}{2} = \frac{3}{2} \\
x = \frac{3}{2}
\]

Replace \( CM \) with \( 4x \) and \( CK \) with \( 2x + 3 \). Subtract 2x from each side. Simplify. Divide each side by 2. Simplify.

Your Turn

d. Suppose \( CK = 5x + 9 \) and \( CM = 6x - 13 \). Find \( x \).

Check for Understanding

1. Complete each statement.
   a. In the same circle, if two chords are \( \text{____} \), then their arcs are congruent.
   b. If a diameter of a circle bisects a chord of the circle, then it is \( \text{____} \) to the chord and bisects its \( \text{____} \).
   c. In a circle, if two \( \text{____} \) are congruent, then their chords are congruent.

2. Refer to \( \bigcirc J \).
   a. Explain why \( \triangle PAT \) is isosceles.
   b. Explain why \( \overline{AG} \cong \overline{TG} \).

Guided Practice

Example 1

Use \( \bigcirc W \) to complete each statement.

3. \( \overline{RS} \cong \text{____} \)
4. \( \overline{ST} \cong \text{____} \)

Example 2

Use \( \bigcirc D \) to find each measure.

5. \( DG \)
6. \( FH \)
7. In \( \odot J \), radius \( JL \) and chord \( MN \) have lengths of 10 centimeters. Find the distance from \( J \) to \( MN \). Round to the nearest hundredth.

8. **Algebra** In \( \odot J \), \( KM \equiv KN \), \( KM = 2x + 9 \), and \( KN = 5x \). Find \( x \), \( KM \), and \( KN \).

---

**Exercises**

**Practice**

Use \( \odot P \) to complete each statement.

9. If \( CD \equiv DE \), then \( \overarc{CD} \equiv \) ?.
10. If \( CD \equiv DE \), then \( \triangle PCD \equiv \triangle \) ?.
11. If \( DP \perp AB \), then \( \overarc{AD} \equiv \) ?.
12. If \( AE \equiv BC \), then \( \overarc{AE} \equiv \) ?.
13. If \( AB \perp CF \), then \( FG \equiv \) ?.
14. If \( AE \equiv BC \), then \( AC \equiv \) ?.

Use \( \odot Q \), where \( QE \perp TN \), to complete each statement.

15. If \( QT = 8 \), then \( QN = \) ?.
16. If \( TE = 6 \), then \( TN = \) ?.
17. If \( TN = 82 \), then \( ET = \) ?.
18. If \( QE = 3 \) and \( EN = 4 \), then \( QN = \) ?.
19. If \( QN = 13 \) and \( EN = 12 \), then \( QE = \) ?.
20. If \( TN = 16 \) and \( QE = 6 \), then \( QN = \) ?.

21. In \( \odot M \), \( RS = 24 \) and \( MW = 5 \). Find \( MT \).

22. In \( \odot A \), \( AL \perp JK \), \( AM = 5 \), and \( AL = 3 \). Find \( JK \).

**Applications and Problem Solving**

23. **Algebra** In \( \odot O \), \( MN \equiv PQ \), \( MN = 7x + 13 \), and \( PQ = 10x - 8 \). Find \( PS \).
24. **Entertainment** In a recent movie, teacher Justin McCleod uses the following method to show his student how to find the center of a circle.

(A man) wants to erect a pole in the center of his circle. But how does he find that center? . . . Draw a circle $ABC$. Draw within it any straight line $AB$. Now bisect (line) $AB$ at $D$ and draw a straight line $DC$ at right angles to (line) $AB$. . . . Okay, (draw) any other straight line. . . $AC$. . . . Bisect (segment) $AC$ (with a perpendicular line) and you get the center of your circle.

a. Draw a figure that matches this description. Assume that “circle $ABC$” means a circle that goes through the points $A$, $B$, and $C$.

b. Explain why this works.

25. **Critical Thinking** In $\triangle ABC$, $E$ and $F$ are diameters.

a. Determine whether $AK > AT$ or $AT > AK$. Why?

b. Name the midpoint of $DG$.

c. Name an arc that is congruent to $GE$.

d. If $K$ is the midpoint of $TD$, is $C$ necessarily the midpoint of $BD$?

26. **Architecture** The Robinsons’ front door has a semicircular window that is divided into four congruent sections, as shown at the right. What is the measure of each arc? (Lesson 11–2)

In the figure, the diameter of the circle is 26 centimeters. (Lesson 11–1)

27. Name two chords.

28. Name three radii.

29. Find $ED$.

30. **Grid In** $\triangle DFG$ is isosceles with vertex angle $F$. Find $m\angle G$. (Lesson 6–4)

31. **Multiple Choice** Eighty-six percent of U.S. adults said that they regularly participated in arts activities in a recent year. Their top five favorite activities are shown in the table below. Which type of graph would best illustrate this information? (Statistics Review)

<table>
<thead>
<tr>
<th>Activity</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>photography</td>
<td>44%</td>
</tr>
<tr>
<td>weaving/needlepoint/handwork</td>
<td>36%</td>
</tr>
<tr>
<td>painting/drawing</td>
<td>33%</td>
</tr>
<tr>
<td>dancing</td>
<td>30%</td>
</tr>
<tr>
<td>musical instrument</td>
<td>28%</td>
</tr>
</tbody>
</table>

Source: National Assembly of Local Arts Agencies, American Council for the Arts

www.geomconcepts.com/self_check_quiz
For everyday meals, the Williams family’s square kitchen table seats four people. On special occasions, however, the sides can be raised to change the square table to a circular one that seats six. When the table’s top is open, its circular top is said to be circumscribed about the square. We also say that the square is inscribed in the circle.

Some regular polygons can be constructed by inscribing them in circles. In Lesson 1–5, you learned to construct a regular hexagon in this way. The following example demonstrates how to construct another regular polygon.

**Example 1**

**Construct a regular quadrilateral.**

- Construct \( \odot P \) and draw a diameter \( \overline{AC} \).
- Construct the perpendicular bisector of \( \overline{AC} \), extending the line to intersect \( \odot P \) at points \( B \) and \( D \).
- Connect the consecutive points in order to form square \( ABCD \).

The construction of a hexagon can be used to discover another property of chords.

**Materials:** compass, ruler

**Step 1** Construct \( \odot P \).

**Step 2** Use the construction in Lesson 1–5 to draw a regular hexagon. Label the consecutive vertices \( A, B, C, D, E, \) and \( F \).
Step 3  Construct a perpendicular line from the center to each chord.

Step 4  Measure the distance from the center to each chord.

**Try These**

1. What is true about the distance from the center of $\bigodot P$ to each chord?
2. From the construction, what is true of $AB, BC, CD, DE, EF$ and $FA$?
3. **Make a conjecture** about the relationship between the measure of the chords and the distance from the chords to the center.

This activity suggests the following theorem.

**Theorem 11–6**

**Words:** In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

**Model:**

**Symbols:**

\[ \overline{AD} \cong \overline{BC} \text{ if and only if } \overline{LP} \cong \overline{PM}. \]

**Example**

**Algebra Link**

In $\bigodot A$, $PR = 2x + 5$ and $QR = 3x - 27$. Find $x$.

The figure shows that $\overline{PR}$ and $\overline{QR}$ are equidistant from the center of the circle. From Theorem 11–6, we can conclude that $\overline{PR} \cong \overline{QR}$.

\[
\begin{align*}
PR &= QR \\
2x + 5 &= 3x - 27 \\
2x + 5 - 2x &= 3x - 27 - 2x \\
5 &= x - 27 \\
5 + 27 &= x - 27 + 27 \\
32 &= x
\end{align*}
\]

**Your Turn**

In $\bigodot O$, $O$ is the midpoint of $\overline{AB}$. If $CR = -3x + 56$ and $ST = 4x$, find $x$.
Check for Understanding

Communicating Mathematics

1. Look for a pattern in the inscribed polygons. What would a polygon with 200 sides look like?

   - equilateral triangle
   - regular pentagon
   - regular hexagon
   - regular octagon

2. Writing Math Compare and contrast the meanings of the terms circumscribed and inscribed.

Guided Practice

Example 1

Example 2

3. Use the construction of a hexagon to construct an equilateral triangle. Explain each step.

4. In \( \odot T \), \( CD = 19 \). Find \( AB \).

5. In \( \odot R \), if \( AB = 2x - 7 \) and \( CD = 5x - 22 \), find \( x \).

6. Carpentry The strongest rectangular beam that can be cut from a circular log is one whose width is 1.15 times the radius of the log. What is the width of the strongest beam that can be cut from a log 6 inches in diameter?

Exercises

Practice

Use a compass and straightedge to inscribe each polygon in a circle. Explain each step.

7. regular octagon

8. regular dodecagon (12 sides)

Use \( \odot P \) to find \( x \).

9. \( AB = 2x - 4 \), \( CD = x + 3 \)
10. \( AB = 3x + 2 \), \( CD = 4x - 1 \)
11. \( AB = 6x + 7 \), \( CD = 8x - 13 \)
12. \( AB = 3(x + 2) \), \( CD = 12 \)
13. \( AB = 2(x + 1) \), \( CD = 8x - 22 \)
14. \( AB = 4(2x - 1) \), \( CD = 10(x - 3) \)
15. Square $MATH$ is inscribed in $\odot P$ with a radius of 12 centimeters.
   a. Find $m\angle HPT$.
   b. Find $TH$.
   c. What kind of triangle is $\triangle PTH$?
   d. Find the distance from $P$ to $HT$.
   e. Are $\overline{AT}$ and $\overline{MA}$ equidistant from $P$?

   **Draw a figure and then solve each problem.**

16. A regular hexagon is inscribed in a circle with a radius of 18 inches. Find the length of each side of the hexagon.

17. In $\odot K$, chord $\overline{AT}$ is 7 units long, and chord $\overline{CR}$ is 3 units long. Which chord is closer to the center of $\odot K$?

Applications and Problem Solving

18. **Architecture** In 1457, the Italian architect Antonio Filarete designed a star-shaped city called Sforzinda. The plan for the city was constructed by inscribing two polygons within a circle. Which two polygons were used?

19. **Mechanical Drawing** Aisha Turner is drawing a plan for a hexagonal patio at the home of a client. She uses the method at the left to construct the hexagon. What is the one distance Ms. Turner needs to know in order to make the hexagon the correct size?

20. **Critical Thinking** To *truncate* means to change one shape into another by altering the corners. So, an octagon is a truncation of a square. Write a paragraph about the relationship between dodecagons and hexagons.

Mixed Review

21. Is it possible to determine whether $\overline{AT} \cong \overline{TB}$? *(Lesson 11–3)*

22. Find $m\angle QTS$ if $m\angle QRS = 50$ and $\overline{ST}$ is a diameter. *(Lesson 11–2)*

23. What is the sum of the measures of the interior angles of a dodecagon (12 sides)? *(Lesson 10–2)*

24. **Grid In** Janine’s kite is a quadrilateral, as shown at the right. What is the perimeter in inches of the interior quadrilateral, assuming the quadrilaterals are similar? *(Lesson 9–7)*

25. **Multiple Choice** On a blueprint, 1 inch represents 10 feet. Find the actual length of a room that is $2\frac{1}{4}$ inches long on the blueprint. *(Lesson 9–2)*

   - A. 20 ft
   - B. 22 ft
   - C. 20$\frac{1}{4}$ ft
   - D. 25 ft

Standardized Test Practice

www.geomconcepts.com/self_check_quiz
A brand of in-line skates advertises “80-mm clear wheels.” The description “80-mm” refers to the diameter of the skates’ wheels. As the wheels of an in-line skate complete one revolution, the distance traveled is the same as the circumference of the wheel.

Just as the perimeter of a polygon is the distance around the polygon, the **circumference** of a circle is the distance around the circle. We can use a graphing calculator to find a relationship between the circumference and the diameter of a circle.

### Graphing Calculator Tutorial

See pp. 782–785.

#### Step 1

Use the Circle tool on the F2 menu to draw a circle.

#### Step 2

Use the Line tool on F2 to draw a line through the center of the circle.

#### Step 3

Use the Segment tool on F2 to draw the segment connecting the two points at which the line intersects the circle.

#### Step 4

Use the Hide/Show tool on F5 to hide the line.

**Try These**

1. Use the Distance & Length tool on F5 to find the circumference of your circle and the length of the diameter.
2. Use the Calculate tool on F5 to find the ratio of the circumference to the diameter.
3. Use the drag key and the arrow keys to make your circle larger. What is the result?
In this activity, the ratio of the circumference $C$ of a circle to its diameter $d$ appears to be a number slightly greater than 3, regardless of the size of the circle. The ratio of the circumference of a circle to its diameter is always fixed and equals an irrational number called $\pi$ or $\pi$. Thus, $\frac{C}{d} = \pi$ or $C = \pi d$. Since $d = 2r$, the relationship can also be written as $C = 2\pi r$.

To the nearest hundredth, the irrational number $\pi$ is approximated by 3.14. The exact circumference is a multiple of $\pi$.

### Examples

**1.** Find the circumference of $\odot O$ to the nearest tenth.

- $C = 2\pi r$ \hspace{1cm} \text{Theorem 11–7}
- $C = 2\pi(3)$ \hspace{1cm} \text{Replace } r \text{ with 3.}
- $C = 6\pi$ \hspace{1cm} \text{This is the exact circumference.}

To estimate the circumference, use a calculator.

Enter: $6 \times \text{2nd} \left[ \pi \right] \text{ ENTER}$

The circumference is about 18.8 centimeters.

**2.** The diameters of a penny, nickel, and quarter are 19.05 millimeters, 21.21 millimeters, and 24.26 millimeters, respectively. Find the circumference of each coin to the nearest millimeter.

<table>
<thead>
<tr>
<th>Penny</th>
<th>Nickel</th>
<th>Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = \pi d$</td>
<td>$C = \pi d$</td>
<td>$C = \pi d$</td>
</tr>
<tr>
<td>$C = \pi(19.05)$</td>
<td>$C = \pi(21.21)$</td>
<td>$C = \pi(24.26)$</td>
</tr>
<tr>
<td>$C \approx 59.84734005$</td>
<td>$C \approx 66.63318018$</td>
<td>$C \approx 76.21503778$</td>
</tr>
</tbody>
</table>

The circumferences are about 60 millimeters, 67 millimeters, and 76 millimeters, respectively.

### Your Turn

a. The circumference of a half dollar is about 96 millimeters. Find the diameter of the coin to the nearest tenth.

b. A circular flower garden has a circumference of 20 feet. Find the radius of the garden to the nearest hundredth.

Data Update: For the latest information about the state quarters that debuted in 1999, visit www.geomconcepts.com
You can use the formula for circumference to solve problems involving figures that have circular arcs.

The 400-meter track at Jackson High School has two straightaways, each 100 meters long, and 2 semicircular ends, each 100 meters around. What is the diameter of each semicircle?

**Explore** You want to know the diameter of each semicircle. You know the length of the semicircular ends.

**Plan** Make a drawing to represent the problem.

**Solve** The two ends together form an entire circle. Since the circumference of a circle is given by \( C = \pi d \), the length \( S \) of a semicircular end can be represented by \( S = \frac{\pi d}{2} \).

\[
S = \frac{\pi d}{2} \quad \text{Original formula}
\]

\[
100 = \frac{\pi d}{2} \quad \text{Replace } S \text{ with } 100.
\]

\[
2 \cdot 100 = 2 \cdot \frac{\pi d}{2} \quad \text{Multiply each side by } 2.
\]

\[
200 = \pi d \quad \text{Simplify.}
\]

\[
\frac{200}{\pi} = \frac{\pi d}{\pi} \quad \text{Divide each side by } \pi.
\]

\[
63.66197724 \approx d \quad \text{Use a calculator.}
\]

The diameter of each semicircle is about 64 meters.

**Examine** Replace \( d \) with 64 in the formula to check the solution.

Enter: \( 64 \times \frac{\pi}{2} \div 2 \) ENTER \( \approx 100.5309649 \)

1. **Explain** why \( C = 2\pi r \) and \( C = \pi d \) are equivalent formulas.

2. **Determine** the exact circumference of the 80-mm diameter wheels described in the lesson.
Guided Practice  

Complete each chart. Round to the nearest tenth.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>d</td>
<td>C</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3.</td>
<td>8 m</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>84.8 ft</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>5 3/4 yd</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>d</td>
<td>C</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4.</td>
<td>2.4 km</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td>32 cm</td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td>3 in.</td>
</tr>
</tbody>
</table>

Example 3  

9. **Law Enforcement** Police officers use a *trundle wheel* to measure skid marks when investigating accidents. The diameter of the wheel is 24 centimeters. What is the distance measured when the wheel makes one complete revolution? Round to the nearest tenth.

Exercises  

Practice  

Find the circumference of each object to the nearest tenth.

- 10. **dime**  
- 11. **top of a can**  
- 12. **bicycle tire**

\[
d = 17.91 \text{ mm} \\
r = 3 \frac{1}{4} \text{ in.} \\
d = 1.7 \text{ m}
\]

Find the circumference of each circle described to the nearest tenth.

- 13. **d = 4 mm**  
- 14. **r = 6 \frac{1}{2} \text{ ft}**  
- 15. **r = 17 \text{ yd}**

Find the radius of the circle to the nearest tenth for each circumference given.

- 16. **47.1 cm**  
- 17. **6.3 \text{ in.}**  
- 18. **18 \text{ km}**  
- 19. The circumference of the top of a tree stump is 8 feet. Find its radius.  
- 20. If the radius of a circle is tripled, how does the circumference change?

Find the circumference of each circle to the nearest hundredth.

- 21.  
- 22.  
- 23.  

**Applications and Problem Solving**

24. **Electronics** Auto speakers are available with 2-inch and 2 \( \frac{5}{8} \)-inch radii. What are the circumferences of the two types of speakers to the nearest tenth?

25. **Bicycling** If the wheels of a bicycle have 24-inch diameters, about how many feet will the bicycle travel when the front wheel makes 200 revolutions?
26. **Gardening** A circular flower bed has a diameter of 5 meters. Different colors of tulip bulbs are to be planted in five equally-spaced concentric circles in the bed. The bulbs will be planted 20 centimeters apart.

   a. What is the diameter of each circle?
   
   b. What is the circumference of each circle?
   
   c. How many bulbs will be needed for each circle?

27. **Critical Thinking** Arcs have degree measure, and they also have length. The length of an arc of a circle is a fractional part of the circumference. Find the length of the arc shown.

**Mixed Review**

28. **Paper Folding** Draw a circle with a 4-inch radius and cut it out. Fold the circle in half. Fold it in half again twice, creasing the edges. Unfold the circle and draw a chord between each pair of adjacent endpoints created by the folds. What figure have you just drawn?  

(Lesson 11–4)

29. **Food** The grill on Mr. Williams’ barbecue is circular with a diameter of 54 centimeters. The horizontal wires are supported by two wires that are 12 centimeters apart, as shown in the figure at the right. If the grill is symmetrical and the wires are evenly spaced, what is the length of each support wire? Round to the nearest hundredth.  

(Lesson 11–3)

30. What is the area of a trapezoid with bases of lengths 36 centimeters and 27 centimeters and a height of 18 centimeters?  

(Lesson 10–4)

31. **Grid In** The measures of four interior angles of a pentagon are 110, 114, 99, and 107. Find the measure of the fifth interior angle.  

(Lesson 10–2)

32. **Multiple Choice** What is the value of $x$ for the pair of congruent triangles?  

(Lesson 5–4)

- A. 45
- B. 23
- C. 11
- D. 6

Quiz 2

**Lessons 11–3 through 11–5**

1. Use $\odot K$ to complete the statement $PQ \equiv \underline{?}$.  

(Lesson 11–3)

2. In $\odot K$, find $x$ if $PQ = 3x - 5$ and $QR = 2x + 4$.  

(Lesson 11–4)

3. In $\odot Q$, $QC = 13$ and $QT = 5$. Find $CB$.  

(Lesson 11–3)

4. Find the circumference of $\odot Q$ to the nearest tenth.  

(Lesson 11–5)

5. Find the radius of a circle whose circumference is 144.5 units.  

(Lesson 11–5)
The space enclosed inside a circle is its area. By slicing a circle into equal pie-shaped pieces as shown below, you can rearrange the pieces into an approximate rectangle. Note that the length along the top and bottom of this rectangle equals the circumference of the circle, $2\pi r$. So, each “length” of this approximate rectangle is half the circumference, or $\pi r$.

The “width” of this approximate rectangle is the radius $r$ of the circle. Recall that the area of a rectangle is the product of its length and width. Therefore, the area of this approximate rectangle is $(\pi r) r$, or $\pi r^2$.

**Example 1**

Find the area of $\odot P$ to the nearest hundredth.

$$A = \pi r^2 \quad \text{Theorem 11–8}$$

$$= \pi (6.3)^2 \quad \text{Replace } r \text{ with } 6.3.$$  

$$= 39.69\pi \quad \text{This is the exact area.}$$

To estimate the area, use a calculator.

**Reading Geometry**

Recall that area is always expressed in square units.

**Your Turn**

a. Find the area of $\odot C$ to the nearest hundredth if $d = 5$ inches.

You can use Theorem 11–8 to find the area of a circle if you know the circumference of the circle.
If $O^\circ A$ has a circumference of $10\pi$ inches, find the area of the circle to the nearest hundredth.

Use the circumference formula to find $r$.

\[
C = 2\pi r \quad \text{Theorem 11–7}
\]
\[
10\pi = 2\pi r \quad \text{Replace } C \text{ with } 10\pi.
\]
\[
\frac{10\pi}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{Divide each side by } 2\pi.
\]
\[
5 = r \quad \text{Simplify.}
\]

To the nearest hundredth, the area is 78.54 square inches.

b. Find the area of the circle whose circumference is 6.28 meters. Round to the nearest hundredth.

You can use the area of a circle to solve problems involving probability.

To win a dart game at a carnival, the dart must land in the red section of the square board. What is the probability that a dart thrown onto the square at random will land in the red section? Assume that all darts thrown will land on the dartboard.

To find the probability of landing in the red section, first subtract the area of the white circle from the area of the large circle.

\[
\text{area of red section} = \text{area of large circle} - \text{area of white circle}
\]
\[
A = \pi r^2 - \pi r^2
\]
\[
= \pi (6^2) - \pi (2^2)
\]
\[
= 36\pi - 4\pi \text{ or } 32\pi
\]

Use a calculator. 32 \(\times\) [2nd] [\(\pi\)] \(\approx\) 100.5309649

The area of the red section is about 101 square inches. The area of the board is $16^2$ or 256 square inches. So, find the probability as follows.

\[
P(\text{landing in the red section}) = \frac{\text{area of the red section}}{\text{area of the board}} \text{ or about } \frac{101}{256}
\]

The probability of landing in the red section is about $\frac{101}{256}$ or 0.395.

In Example 3, we used theoretical probability. The solution was based on the formulas for the areas of a circle and a square. This is different from experimental probability, in which the probability is calculated by repeating some action. To find the experimental probability for this situation, you would have to throw darts at a board like the one described above many times and record how many times the red section was hit and how many times it was not.
Consider the circle at the right. The radius of \( \odot C \) is 14 centimeters, and central angle \( \angle ACB \) has a measure of 90. The shaded region is called a **sector** of the circle. A sector of a circle is a region bounded by a central angle and its corresponding arc. The sector shown is a 90° sector.

Since the sum of the measures of the central angles of a circle is 360, the arc of the sector in \( \odot C \) represents \( \frac{90}{360} \) or \( \frac{1}{4} \) of the circle. Therefore, the area of the sector is \( \frac{1}{4} \) the area of the circle.

\[
\text{Area of } \odot C = \pi r^2 = \pi (14)^2 = 196\pi \text{ cm}^2
\]

\[
\text{Area of sector bounded by } \angle ACB = \frac{1}{4} (\text{area of } \odot C) = \frac{1}{4} (196\pi) = 49\pi \text{ cm}^2
\]

**Theorem 11–9**

**Area of a Sector of a Circle**

If a sector of a circle has an area of \( A \) square units, a central angle measurement of \( N \) degrees, and a radius of \( r \) units, then

\[
A = \frac{N}{360}(\pi r^2).
\]

---

**Example 4**

Find the area of the shaded region in \( \odot P \) to the nearest hundredth.

\[
A = \frac{N}{360}(\pi r^2) \quad \text{Theorem 11–9}
\]

\[
= \frac{150}{360}[\pi (10)^2] \quad \text{Substitution}
\]

\[
= \frac{150}{360}(100\pi) = 10^2 = 100
\]

Enter: 150 ÷ 360 × 100 × [π] ENTER

The area of the shaded region in \( \odot P \) is 130.90 square inches.

**Your Turn**

c. Find the area of a 72° sector if the radius of the circle is \( \frac{7}{3} \) feet. Round to the nearest hundredth.

---

**Check for Understanding**

**Communicating Mathematics**

1. **Show** that Theorem 11–9 verifies the area of the sector for \( \odot C \) at the top of this page.

2. **Writing Math.** Write a convincing argument about whether the circumference and area of a circle could have the same numeric value.
Guided Practice

Complete the chart. Round to the nearest hundredth.

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>d</th>
<th>C</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.75 m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>14.14 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>21.77 in.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 4

6. In a circle with a radius of 9 inches, find the area of the sector whose central angle measures 90. Round to the nearest hundredth.

Example 3

7. **Probability** Assume that all darts thrown will land on the dartboard at the right. Find the probability that a randomly-thrown dart will land in the red region.

Exercises

Find the area of each circle described to the nearest hundredth.

8. \( r = 4 \text{ mm} \)
9. \( r = \frac{11}{2} \text{ ft} \)
10. \( r = 6.7 \text{ cm} \)

11. \( d = 15 \text{ mi} \)
12. \( d = \frac{11}{3} \text{ in.} \)
13. \( d = 4.29 \text{ m} \)

14. \( C = 81.68 \text{ cm} \)
15. \( C = 37.07 \text{ m} \)
16. \( C = 14\frac{3}{4} \text{ ft} \)

17. Find the area of a circle whose diameter is 18 centimeters. Round to the nearest hundredth.

18. What is the radius to the nearest hundredth of a circle whose area is 719 square feet?

In a circle with radius of 6 centimeters, find the area of a sector whose central angle has the following measure.

19. 20
20. 90
21. 120

22. Find the area to the nearest hundredth of a 10° sector in a circle with diameter 12 centimeters.

23. The area of a 60° sector of a circle is 31.41 square meters. Find the radius of the circle.

Assume that all darts thrown will land on a dartboard. Find the probability that a randomly-thrown dart will land in the red region. Round to the nearest hundredth.

24.

![Diagram of a circle with a sector shaded red.]

25.
26. **Biology**  About 1.5 million species of animals have been named thus far. The circle graph shows how the various groups of named animals compare. If the sector representing insects is a 240° sector and the radius is \( \frac{3}{4} \) inch, what is the area of that sector?

27. **Cooking**  When Julio bakes a pie, he likes to put foil around the edges of the crust to keep it from getting too brown. He starts with a 12-inch square of foil, folds it in fourths, and tears out a sector with a radius of 4 inches. Then he places it over the pie. What is the area of the remaining piece of foil to the nearest hundredth?

28. **Critical Thinking**  Refer to the circle at the right.
   a. Find the area of the shaded region to the nearest hundredth if \( r = 2, 3, 4, 5, 6, \) and 8 inches.
   b. What is the relationship between the area of the shaded region and the area of the large circle?
   c. **Probability**  Suppose this figure represents a dartboard. What is the probability that a randomly-thrown dart will land in the yellow region?

29. **Animals**  Taylor is building a circular dog pen for her new puppy. If the diameter of the pen will be 12 meters, about how many meters of fencing will Taylor need to purchase?  \((Lesson 11–5)\)

30. In \( \odot P \), if \( JK = 3x - 4 \) and \( LM = 2x + 9 \), find \( x \).  \((Lesson 11–4)\)

31. **Construction**  Jonathan Werner is building a deck shaped like a regular octagon with an apothem 7.5 feet long and sides each 6.2 feet long. If wood for the deck floor costs $1.75 per square foot, how much will it cost Mr. Werner to install the deck floor?  \((Lesson 10–5)\)

32. **Short Response**  Use the figure to complete the following statement.  \((Lesson 9–6)\)
   \[
   \frac{AC}{BC} = \frac{?}{ED}
   \]

33. **Multiple Choice**  Which angle forms a linear pair with \( \angle SVT \)?  \((Lesson 3–4)\)
   - A \( \angle RVS \)
   - B \( \angle PVQ \)
   - C \( \angle RVQ \)
   - D \( \angle PVT \)
Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

adjacent arcs (p. 463)
arc (p. 462)
center (p. 454)
central angle (p. 462)
chord (p. 454)
circle (p. 454)
circumference (p. 478)
circumscribed (p. 474)
concentric (p. 456)
diameter (p. 454)
experimental probability (p. 484)
inscribed (p. 474)
loci (p. 460)
locus (p. 460)
major arc (p. 462)
minor arc (p. 462)
pi (p. 479)
radius (p. 454)
sector (p. 485)
semicircle (p. 462)
theoretical probability (p. 484)

State whether each sentence is true or false. If false, replace the underlined word(s) to make a true statement.

1. A central angle is an angle whose vertex is at the center of the circle and whose sides intersect the circle.
2. A chord that contains the center of a circle is called a radius.
3. The diameter of a circle is the distance around the circle.
4. pi is the ratio of the circumference of a circle to its diameter.
5. A sector is a segment whose endpoints are the center of the circle and a point on the circle.
6. A chord is a segment whose endpoints are on the circle.
7. A semicircle is an arc whose endpoints are on the diameter of a circle.
8. A sector of a circle is a region bounded by a central angle and its corresponding center.
9. A major arc is the part of the circle in the exterior of the central angle.
10. Two circles are inscribed if they lie in the same plane, have the same center, and have radii of different lengths.

Skills and Concepts

Objectives and Examples

- Lesson 11–1 Identify and use parts of circles.

\[
LM \text{ is a radius of } \odot M. \\
\text{If } LM = 16, \text{ find } JK.
\]

\[
d = 2r \\
JK = 2(LM) \quad \text{Substitution} \\
JK = 2(16) \text{ or } 32 \quad \text{Substitution}
\]

Review Exercises

Refer to the circle at the left to complete each statement in Exercises 11–13.

11. \( KN \) is a \( \_\_\_\_\_\_ \) of \( \odot M \).
12. A diameter of \( \odot M \) is \( \_\_\_\_\_\_ \).
13. \( JM, KM, \) and \( LM \) are \( \_\_\_\_\_\_ \) of \( \odot M \).

14. Find the measure of diameter \( AC \)
if \( BP = x \) and \( AC = 5x - 6 \).
**Objectives and Examples**

**Lesson 11–2** Identify major arcs, minor arcs, and semicircles and find the measures of arcs and central angles.

In \( \odot S \), find \( m\overline{UV} \) and \( m\overline{TUV} \).

\[
\begin{align*}
m\overline{UV} &= m\angle USV \\
&= 65 \\
m\overline{TUV} &= 360 - m\angle VST \\
&= 360 - 115 \text{ or } 245
\end{align*}
\]

**Lesson 11–3** Identify and use the relationships among arcs, chords, and diameters.

In \( \odot Q \), if \( \overline{QU} \perp \overline{RT} \) and \( RT = 18 \), find \( RS \).

\[
\begin{align*}
RT &= 2(RS) \\
18 &= 2(RS) \quad \text{Substitution} \\
\frac{18}{2} &= \frac{2(RS)}{2} \quad \text{Divide each side by 2.} \\
9 &= RS
\end{align*}
\]

**Lesson 11–4** Inscribe regular polygons in circles and explore the relationship between the length of a chord and its distance from the center of the circle.

In \( \odot P \), \( RT = 52 \). Find \( XY \).

\[
\begin{align*}
RT &= XZ & \text{Theorem 11–6} \\
RT &= 2(XY) & \text{Theorem 11–5} \\
52 &= 2(XY) & \text{Substitution} \\
\frac{52}{2} &= \frac{2(XY)}{2} & \text{Divide each side by 2.} \\
26 &= XY
\end{align*}
\]

**Review Exercises**

Find each measure in \( \odot T \) if \( \overline{PQ} \) is a diameter.

15. \( m\overline{PQS} \)
16. \( m\angle QTS \)
17. \( m\overline{PS} \)

\( B \) is the center of two circles with radii \( BC \) and \( BD \). If \( m\angle DBJ = 113 \) and \( KL \) and \( JM \) are diameters, find each measure.

18. \( m\overline{CK} \)
19. \( m\overline{DMJ} \)
20. \( m\overline{CL} \)

Use \( \odot X \) to complete each statement.

21. If \( AC = 12 \), then \( CD = \_\_\_\_\_\_\_\_\_ \).
22. If \( DX = 18 \) and \( AC = 48 \), then \( CX = \_\_\_\_\_\_\_\_\_ \).

23. In \( \odot L \), \( \overline{MQ} \cong \overline{NQ} \). If \( MN = 16 \) and \( LN = 10 \), find \( LQ \).

24. In \( \odot C \), \( TU = 12x - 7 \) and \( TV = 3x + 20 \). Find \( x \).

Use \( \odot C \) to determine whether each statement is *true* or *false*.

25. \( \overline{AC} \cong \overline{BC} \)
26. \( \overline{AU} \cong \overline{BV} \)
27. \( \angle ATB \cong \angle ACB \)
28. \( \overline{BT} \cong \overline{BV} \)
• **Lesson 11–5**  Solve problems involving circumferences of circles.

Find the circumference of \( \odot Q \) to the nearest hundredth.

\[
C = 2\pi r \\
C = 2\pi (5) \\
C = 10\pi \text{ or about } 31.42
\]

The circumference is about 31.42 meters.

---

• **Lesson 11–6**  Solve problems involving areas and sectors of circles.

Find the area of \( \odot T \) to the nearest tenth.

\[
A = \pi r^2 \\
A = \pi (12.9)^2 \\
A = 166.41\pi \text{ or about } 522.8
\]

The area is about 522.8 square yards.

---

**Applications and Problem Solving**

39. **Cycling**  Suppose a bicycle wheel has 30 spokes evenly spaced and numbered consecutively from 1 through 30. Find the measure of the central angle formed by spokes 1 and 14.  \((\text{Lesson } 11–2)\)

40. **Swimming**  Swimmers often use kickboards when they want to concentrate on their kicking. Find the width of the kickboard shown below.  \((\text{Lesson } 11–5)\)

41. **Food**  At a school function, a fruit pizza was served that had a diameter of 14 inches and was cut into 10 equal-sized wedges. By the end of the evening, 7 consecutive wedges had been eaten. Find the area of the remaining pizza to the nearest square inch.  \((\text{Lesson } 11–6)\)
1. Define *inscribed polygon* and give an example of one in everyday life.

2. Explain how the perimeter of a polygon and the circumference of a circle are related.

**Use \( \odot L \) to complete each statement.**

3. \( \overline{MP} \) is a ____ of \( \odot L \).
4. If \( MN = 16 \), then \( LN = \) ____.
5. All radii of a circle are ____.

**Find each measure in \( \odot L \) if \( m\angle NLQ = 79 \).**

6. \( m\overline{NQ} \)
7. \( m\overline{MNQ} \)
8. \( m\angle MLQ \)

**Use \( \odot W \) to complete each statement.**

9. If \( XZ = 18 \), then \( YZ = \) ____.
10. If \( WY = 12 \) and \( XY = 16 \), then \( XW = \) ____.
11. If \( XZ = 16 \) and \( WY = 6 \), then \( WZ = \) ____.
12. In \( \odot B \), \( CD = 62 \). Find \( AE \).

**Find the circumference of each circle described to the nearest tenth.**

13. \( r = 14.3 \text{ in.} \)
14. \( d = 33 \text{ m} \)
15. \( r = 27 \text{ ft} \)

**Find the area of each circle to the nearest hundredth.**

16. \[ \begin{array}{c} 7.6 \text{ yd} \\ \end{array} \]
17. \[ \begin{array}{c} 53 \text{ cm} \\ \end{array} \]
18. \[ \begin{array}{c} 1.4 \text{ km} \\ \end{array} \]

19. **Algebra** In \( \odot S \), \( JL = 12x + 2 \) and \( MP = 3x + 20 \). Find \( x \).

20. **Gardening** Eloy is preparing to plant flowers in his circular garden. The garden has a diameter of 4 meters and is divided into six equal portions. If he intends to fill the shaded portions of the garden with marigolds, what is the area that will remain for other types of flowers?
**Example 1**

The graph of a line is shown at the right. What is the equation of the line?

- **A** \( y = -\frac{1}{4}x - 2 \)
- **B** \( y = \frac{1}{4}x - 2 \)
- **C** \( y = 4x - 2 \)
- **D** \( y = -4x - 2 \)

**Solution**

Locate two points on the graph. The \( y \)-intercept is at \((0, -2)\). Another point is at \((1, 2)\). Find the slope of the line, using these two points. Notice that the \( x \)-coordinate increases by 1 and the \( y \)-coordinate increases by 4. Therefore, the slope is \( \frac{4}{1} \) or 4. Choose the equation with a slope of 4. The answer is **C**.

Check your answer by replacing \( x \) and \( y \) with the coordinates of the point \((0, -2)\) and see if the statement is true. Do the same for \((1, 2)\).

\[
\begin{align*}
y &= 4x - 2 \\
-2 &= 4(0) - 2 \\
-2 &= -2 & \checkmark
\end{align*}
\]

**Hint**

Look carefully at the answer choices.

**Example 2**

If \( x \otimes y = \frac{1}{x - y} \), what is the value of \( \frac{1}{2} \otimes \frac{1}{3} \)?

- **A** \(6\)
- **B** \(\frac{6}{5}\)
- **C** \(\frac{1}{6}\)
- **D** \(-1\)
- **E** \(-6\)

**Solution**

Substitute \( \frac{1}{2} \) for \( x \) and \( \frac{1}{3} \) for \( y \) in the expression. Then simplify the fraction.

\[
\begin{align*}
\frac{1}{2} \otimes \frac{1}{3} &= \frac{1}{\frac{1}{2} - \frac{1}{3}} & \text{Substitution} \\
&= \frac{1}{\frac{3}{6} - \frac{2}{6}} & \text{The LCD is 6.} \\
&= \frac{1}{\frac{1}{6}} & \text{Subtraction} \\
&= 6 & 1 \div \frac{1}{6} = 1 \times 6
\end{align*}
\]

The answer is **A**.
After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

**Multiple Choice**

1. The table shows how many sit-ups Luis can do after a number of weeks. How many sit-ups will he be able to do after 7 weeks?  
*(Algebra Review)*

<table>
<thead>
<tr>
<th>Week</th>
<th>Number of Sit-ups</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
</tr>
</tbody>
</table>

2. In the triangle at the right, what is \( m \angle A \)?  
*(Lesson 5–2)*

\[ 3x \] \[ 2x \]

\[ \triangle ABC \]

3. Which equation best describes the graph?  
*(Lesson 4–6)*

\[ a. \ y = x - 1 \]
\[ b. \ y = -x - 4 \]
\[ c. \ y = -3x + 1 \]
\[ d. \ y = \frac{1}{3}x + 1 \]

4. For all integers \( Z \), suppose \( \Delta = Z^2 \) (if \( Z \) is odd), and \( \sqrt{Z} \) (if \( Z \) is even). What is the value of \( \Delta + \sqrt{Z} \)?  
*(Algebra Review)*

\[ a. \ 9 \]
\[ b. \ 15 \]
\[ c. \ 39 \]
\[ d. \ 87 \]

5. A plumber charges $75 for the first 30 minutes of each house call plus $2 for each additional minute. She charged Mr. Adams $113. For how long did the plumber work?  
*(Algebra Review)*

\[ a. \ 38 \text{ min} \]
\[ b. \ 44 \text{ min} \]
\[ c. \ 49 \text{ min} \]
\[ d. \ 59 \text{ min} \]
\[ e. \ 64 \text{ min} \]

6. Which equation represents the relationship shown in the table?  
*(Algebra Review)*

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

\[ a. \ y = 3x - 2 \]
\[ b. \ y = 2x + 1 \]
\[ c. \ y = 2x^2 - x \]
\[ d. \ y = x^2 - 2 \]

7. What is \( m \angle PQR \)?  
*(Lesson 7–2)*

\[ 40^\circ \]
\[ 55^\circ \]
\[ 70^\circ \]
\[ 110^\circ \]

8. If 10% of \( x \) is 20% of 100, what is \( x \)?  
*(Percent Review)*

\[ a. \ 200 \]
\[ b. \ 100 \]
\[ c. \ 20 \]
\[ d. \ 5 \]

**Grid In**

9. What number should come next in this sequence:  
\[ 17, 31, 7, 25, \ldots \]  
*(Algebra Review)*

\[ a. \ 4' \]
\[ b. \ 8' \]
\[ c. \ 2' \]
\[ d. \ 8' \]

**Extended Response**

10. | Speed (mph) | Distance (ft) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>289</td>
</tr>
<tr>
<td>60</td>
<td>332</td>
</tr>
<tr>
<td>65</td>
<td>378</td>
</tr>
<tr>
<td>70</td>
<td>426</td>
</tr>
</tbody>
</table>

Part A Graph the ordered pairs (speed, distance) on a coordinate plane.  
*(Algebra Review)*

Part B Describe how stopping distance is related to speed.